Academic Year: 2015 – 2016 Semester: Spring Date: May 5, 2016	Modern University for Technology & Information مستقبل الصفوة **Faculty of Pharmacy**	Mathematics: OCM 103 Final Exam Duration Time: **2** Hours
Answer All Questions		No. of questions: 4 Total Mark: 60

Question 1

(a)If $A = \begin{bmatrix} 1 & 2 & -2 \\ 0 & 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 2 & 1 \end{bmatrix}$ — 10

Find, if possible, $A + B$, $A.A$, $A.B$, $A.B^t$, $|A|$, $|A^t.B|$.

(b)Find the eigenvalues and eigenvectors of : $A = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$. — 6

Question 2

(a)Find **y`** where: — 12

(i) $y = x^3 + 3^x + 3x$ (ii) $y = x^2 . 2^x + 4$ (iii) $y = \cos x . \log x$

(iv) $y = [x^3 - \sin x]^6$ (v) $y = 3 + \sin^5 x$ (vi) $y = \frac{2}{x^2} + \frac{\ln x}{2x+1}$

(b)Find the integrals: — 12

(i) $\int (x^4 + 2^x) dx$ (ii) $\int \left(\frac{2}{3} + \frac{1}{x^3}\right) dx$ (iii) $\int (2\cos x - \sin x)\, dx$

(iv) $\int (\sqrt{x} + e^x)\, dx$ (v) $\int x \cos x\, dx$ (vi) $\int_0^1 (x^2 + 2)^2\, dx$

Question 3

(a)Find the extrema of the function : $f(x) = x^3 - 6x^2 + 2$ — 5

(b)If a drug exists in three dosage forms : The first of concentration 1 mg / tablet , The second of concentration 2 mg / tablet , The third of concentration 4 mg /tablet. If the pharmacist wanted to produce 8 tablets of concentration 2.5 mg / tablet by mixing whole tablets. Find two possible solutions. — 5

Question 4

(a)If y is the quantity of drug decreases according to the equation $\frac{dy}{dt} = -y^{\frac{1}{2}}$. — 5

Find y as function of the time t where the initial quantity is 16 units.

Also, find (i)The value of y after 2 hours.

(ii)The time at which there is no drug in the blood.

(b)If the quantity of a drug in the blood decreases according to the data: — 5

Time: t	0	2	4	6	8	10	Hours
Quantity: y	20	19	16	11	5	1	Units

From these data, find the relation $y = a + bt$.

Good Luck *Dr. Mohamed Eid*

Answer

Question 1

(a) A.A , A.B , |A| are not exist.

$$A+B=\begin{bmatrix}1&2&-2\\0&3&-1\end{bmatrix}+\begin{bmatrix}2&0&1\\1&2&1\end{bmatrix}=\begin{bmatrix}3&2&-1\\1&5&0\end{bmatrix}$$

$$A.B^t=\begin{bmatrix}1&2&-2\\0&3&-1\end{bmatrix}\begin{bmatrix}2&1\\0&2\\1&1\end{bmatrix}=\begin{bmatrix}0&3\\-1&5\end{bmatrix}$$

$$A^t.B=\begin{bmatrix}1&0\\2&3\\-2&-1\end{bmatrix}\begin{bmatrix}2&0&1\\1&2&1\end{bmatrix}=\begin{bmatrix}2&0&1\\7&6&5\\-5&-2&-3\end{bmatrix}$$

$$|A^t.B|=2(-18+10)-0+(-14+30)=0$$

---*10-Marks*

(b) $|A-\lambda I|=\begin{vmatrix}2-\lambda&4\\1&2-\lambda\end{vmatrix}$

$$=(2-\lambda)(2-\lambda)-4=\lambda^2-4\lambda=0$$

Then, the eigenvalues are: $\lambda_1=0,\quad \lambda_2=4.$

From the equation, $\begin{bmatrix}2-\lambda&4\\1&2-\lambda\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=0$

For : $\lambda_1=0$, $\begin{bmatrix}2&4\\1&2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=0$

Then 2x + 4y = 0, 2x + 4y = 0

Then x = – 2y = any number except 0

Put y = 1, we get x = –2 and the corresponding eigenvector is:

$X_1=\begin{bmatrix}-2\\1\end{bmatrix}$

For : $\lambda_2=4$, $\begin{bmatrix}-2&4\\1&-2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=0$

Then –2x + 4y = 0, x – 2y = 0

Then x = 2y = any number except 0

Put y = 1, we get x = 2 and the corresponding eigenvector is:

$X_2=\begin{bmatrix}2\\1\end{bmatrix}$

-- *(6 Marks)*

Question 2

(a)(i) $y=x^3+3^x+3x$, $\quad y`=3x^2+3^x.\ln 3+3$

(ii) $y=x^2.2^x+4$, $\quad y`=x^2.2^x.\ln 2+2x.2^x$

(iii) $y=\cos x.\log x$, $\quad y`=\cos x.\frac{1}{\ln 10}.\frac{1}{x}-\sin x.\log x$

(iv) $y=[x^3-\sin x]^6$, $\quad y`=6[x^3-\sin x]^5.[3x^2-\cos x]$

(v) $y = 3 + \sin^5 x$, $y` = 0 + 5[\sin x]^4 . \cos x$

(vi) $y = \frac{2}{x^2} + \frac{\ln x}{2x+1}$, $y` = -4x^{-3} + \frac{(2x+1).\frac{1}{x} - 2\ln x}{(2x+1)^2}$

--- *(12 Marks)*

(b)(i) $\int (x^4 + 2^x) dx = \frac{1}{5} x^5 + \frac{1}{\ln 2} 2^x + c$

(ii) $\int (\frac{2}{3} + \frac{1}{x^3})\, dx = \frac{2}{3} x - \frac{1}{2} x^{-2} + c$

(iii)$\int (2\cos x - \sin x)\, dx = 2\sin x + \cos x + c$

(iv) $\int (\sqrt{x} + e^x)\, dx = \frac{2}{3} x^{3/2} + e^x + c$

(v)$\int x \cos x\, dx = x \sin x + \cos x + c$, By parts.

(vi)$\int_0^1 (x^2 + 2)^2\, dx = \int_0^1 (x^4 + 4x^2 + 4)\, dx = \frac{1}{5} x^5 + \frac{4}{3} x^3 + 4x = \frac{83}{15}$

--- *(12 Marks)*

Question 3

(a)Since $f(x) = x^3 - 6x^2 + 2$, $f`(x) = 3x^2 - 12x = 0$

Then $x^2 - 4x = x(x - 4) = 0$. Then, the critical points are 0, 4.

From $f``(x) = 6x - 12$.

Then $f``(0) = 0 - 12 = -12$, then 0 is maximum.

$f``(4) = 24 - 12 = 12$, then 4 is minimum.

---*5-Marks*

(b) This problem can be formulated in mathematical model as:
Assume that x = number of tablets taken from the first type
y = number of tablets taken from the second type
z = number of tablets taken from the third type
Then x + y + z = 8, x + 2y + 4z = 8x2.5 = 20, x, y, z ≥ 0, integers
It is a system of linear equations with integrality conditions.
This system can be solved as:
Rewrite this system as: y + z = 8 – x (i)
2y + 4z = 20 – x (ii)
Multiply equation (i) by – 4 and add to equation (ii) to eliminate z.
We get – 2y = – 32 + 4x + 20 – x = –12 + 3x. Then $y = 6 - \frac{3}{2} x$.
From equation (i), $y = 2 + \frac{1}{2} x$
Then, the solution of the system is given by:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ 6 - \frac{3}{2}x \\ 2 + \frac{1}{2}x \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 2 \end{bmatrix}, \qquad \begin{bmatrix} 1 \\ 9/2 \\ 5/2 \end{bmatrix}, \qquad \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix}$$

Acceptable rejected acceptable

--*5-Marks*

<u>Question 4</u>

(a)From the equation : $\frac{\mathrm{dy}}{\mathrm{dt}} = -\mathrm{y}^{\frac{1}{2}}$, then $\mathrm{y}^{-\frac{1}{2}}\mathrm{dy} = -\mathrm{dt}$

Integrate both sides, we get $\frac{\mathrm{y}^{\frac{1}{2}}}{\frac{1}{2}} = -\mathrm{t} + \mathrm{c}$ Or $2.\sqrt{\mathrm{y}} = -\mathrm{t} + \mathrm{c}$

At t = 0 , y = 16, we get $8 = 0 + \mathrm{c}$.

Then c = 8 and $2.\sqrt{\mathrm{y}} = -\mathrm{t} + 8$.

Then $\sqrt{y} = \frac{1}{2}(8 - \mathrm{t})$. The explicit relation is : $\mathrm{y} = \frac{1}{4}(8 - \mathrm{t})^2$

(i)After 2 hours, we get $\mathrm{y} = \frac{1}{4}(8 - 2)^2 = \frac{36}{4} = 9$ units.

(ii) There is no drug in the blood when y = 0, then $0 = \frac{1}{4}(8 - \mathrm{t})^2$.

Then, the time is 8 hours.

--*5-marks*

(b)Using Calculator, we get the line :

$$\mathrm{y} = \mathrm{a} + \mathrm{bt} = 22.14 - 2.03\mathrm{t}$$

--*5-marks*